Exam. Code : 103204
Subject Code : 1120

B.A./B.Sc. 4th Semester

MATHEMATICS

Paper—I

(Statics & Vector Calculus)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Do any *five* questions, selecting at least *two* questions from each section. All questions carry equal marks.

SECTION—A

1. (a) If two forces P and Q act along OA and OB and their resultant meets the line AB in the point C, find the position of the point C in which their resultant cuts AB.

(b) The ends of an inelastic string 0.17 m long are attached to two points A and B, 0.13 m apart in the same horizontal line. A weight of 4 kg. is attached to the point O of the string 0.05 m from end A. Find the tension in each portion of the string.

2. (a) P and Q are magnitudes of two like parallel forces. If first force be moved parallel to itself through a distance x, show that their resultant moves through a distance $\frac{Px}{P+Q}$.

(b) Apply Varignon's theorem to find the moment of a force of 200 kg. wt. lying in the XY-plane and acting at the point (1, 2) and directed away from the origin O about the origin O. The force makes an angle of 30° with the X-axis.

3. (a) Three like parallel forces of magnitude 2P + Q, 4P – Q and 8 Newton act at the vertices of a triangle. Find P and Q if the resultant of the three parallel forces passes through the centroid of the triangle.

(b) ABCDEF is a regular hexagon. Forces P, 2P, 3P, 2P, 5P, 6P act along AB, BC, DC, ED, EF and AF, respectively. Show that the six forces are equivalent to a couple and find its moment.

4. (a) Masses 2, 4, 6, 5, x, y kgs. are placed at the corners A, B, C, D, E, F of a regular hexagon. Find the values of x and y so that the C.G. coincides with the centre of hexagon.

(b) A ladder of weight W rests with one end against a smooth vertical wall and with the other resting on a smooth floor. If the inclination of the ladder to the horizon be 60°, find the horizontal force that must be applied to the lower end to prevent the ladder from sliding down.

5. (a) A body is placed on a rough plane inclined to the horizon at an angle greater than the angle of friction and is supported by a force acting parallel to the plane and along a line of greatest slope. Find the limits between which the force must lie.

(b) If the force which acting parallel to a rough plane of inclination α to the horizon is just sufficient to draw a weight up by n times the force which will just be on the point of sliding down, show that the $\tan\alpha = \mu\frac{n+1}{n-1}$.

SECTION—B

6. (a) Prove that the necessary and sufficient condition for the vector function $\vec{f}(t)$ to have constant direction is $\vec{f}\times\frac{d\vec{f}}{dt}=\vec{0}$.

(b) Show that :

(i) $\nabla(\vec{r}\cdot\vec{a})=\vec{a}$

(ii) $\nabla[\vec{r},\vec{a},\vec{b}]=\vec{a}\times\vec{b}$, where $\vec{a}$ and $\vec{b}$ are constant vectors.

7. (a) Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at the point (1, –2, 1) in the direction of $2\hat{i}-\hat{j}-2\hat{k}$.

(b) If $\vec{a}$ is a constant vector, then show that

$$\nabla \times \left(\frac{\vec{a} \times \vec{r}}{r^n} \right) = \frac{2-n}{r^n} + \frac{n(\vec{a} \cdot \vec{r})}{r^{n+2}} \vec{r}.$$

8. (a) Verify divergence theorem for $\vec{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

(b) Verify Green's theorem in the plane for $\oint_C [(xy + y^2)dx + x^2dy]$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

9. (a) By transforming to triple integral evaluate

$$I = \int\int_S (x^3 dydz + x^2 ydzdx + x^2 zdxdy),$$

where S is the closed surface bounded by the plane $z = 0$, $z = b$ and the cylinder $x^2 + y^2 = a^2$.

(b) If $\vec{r} \times \overrightarrow{dr} = \vec{0}$ show that $\hat{r}$ = constant.

10. (a) State and prove Stoke's theorem.

(b) Apply Green's theorem in plane to evaluate $\oint_C [(y - \sin x)dx + \cos x dy]$, where C is the triangle enclosed by the lines $y = 0$, $2x = \pi$, $\pi y = 2x$.